

THE TWO-PHOTON LASER

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The stationary regime of a two-photon laser and its stability are studied. Resonance curves for the excitation of stationary oscillations are compiled. The non-stationary processes in this system are investigated in the approximation of the given field. When the excitation conditions are fulfilled, the laser emits an impulse having the frequency ω_1 (frequency of the operating transition $\omega_{21} = \omega_1 + \omega_2$, where ω_2 is the field frequency $E_2 > E_2^{\text{thresh}}$ from an outer source).

The impulse strength and its duration depend essentially on the magnitude of the field E_2 . If $\frac{E_2}{E_2^{\text{thresh}}} \geq 1$, then

the emitted impulse has a symmetrical form and its energy and duration are determined by the field E_2 . In the case of $\frac{E_2}{E_2^{\text{thresh}}} \gg 1$, the signal energy depends only

on the properties of the active medium. The duration of the leading edge is determined by the magnitude of the field E_2 ; the duration of the trailing edge equals the damping time τ_1 of the resonator at the frequency ω_1 .

Initial Equations. Stationary Regime of a Two-Photon Laser

/39*

The basic equations describing non-stationary processes in a two-photon laser were obtained in (Ref. 1-3). With allowance for the phase relationships, these equations have the following form (Ref. 3)

$$\begin{aligned}
 \frac{dP_3}{dt} + \frac{P_3}{T_{12}} &= -\frac{2\rho_{12}}{\hbar} a E_1 E_2 N_{21} \sin \Phi, \\
 \frac{dE_1}{dt} &= -\frac{\omega_1}{2Q_1} E_1 - 4\pi\rho_{12}\omega_1 a E_2 P_3 \sin \Phi, \\
 \frac{dE_2}{dt} &= -\frac{\omega_2}{2Q_2} E_2 - 4\pi\rho_{12}\omega_2 a E_1 P_3 \sin \Phi, \\
 \frac{d\Phi}{dt} &= \Lambda + \delta N_{21} - \frac{2\rho_{12}}{\hbar} a N_{21} \frac{E_1 E_2}{P_3} \cos \Phi - \\
 &\quad - 4\pi\rho_{12}a \left(\omega_1 \frac{E_2}{E_1} + \omega_2 \frac{E_1}{E_2} \right) P_3 \cos \Phi, \\
 \frac{dN_{21}}{dt} + \frac{N_{21} - N_{21}^0}{T_1} &= \frac{8\rho_{12}}{\hbar} a E_1 E_1 P_3 \cos \Phi + 2A(E_1^2 + E_2^2) - 2B(E_1^2 + E_2^2) N_{21},
 \end{aligned} \tag{1}$$

* Numbers in the margin indicate pagination in the original foreign text.

where E_1 and E_2 are the field amplitudes which change slowly with time (as compared with $e^{i\omega_n t}$, $n = 1, 2$) in a resonator at the frequencies ω_1 and ω_2 , respectively. The frequency of the operational transition ω_{21} is related to the eigen frequencies of the resonator by the relationship $\omega_{21} = \omega_1 + \omega_2$; P_3 - polarization of the active substance located in the resonator; $\Phi = \phi_1 + \phi_2 - \psi$ - phase difference between fields in the resonator and the polarization of the substance; Q_1, Q_2 - quality of the resonator at the frequencies ω_1, ω_2 ; N_{21} - population difference between the operational levels; the quantities N_{21}^0 and T_1 are related to the transition probability by the following relationships:

$$N_{21}^0 = \frac{W_{13}W_{32} - W_{21}(W_{31} + W_{32})}{W_{13}W_{32} + W_{21}(W_{31} + W_{32})} N_0,$$

$$T_1 = \frac{W_{31} + W_{32}}{W_{13}W_{32} + W_{21}(W_{31} + W_{32})}.$$

where p_{mn} are the matrix elements of the dipole moment; T_{mn} - relaxation time of the corresponding, non-diagonal density matrix element; the frequency difference $\Delta = \frac{\omega_{10}^2 - \omega_1^2}{2\omega_1} + \frac{\omega_{20}^2 - \omega_2^2}{2\omega_2} - 2\pi\omega_{21} \frac{p_{13}^2 + p_{23}^2}{\hbar\omega_{31}} N_0, N_0$ - number of particles per unit volume of the active substance

$$a = \frac{p_{13}p_{23}}{\hbar\omega_{31}p_{12}}, \quad A = \frac{2}{\hbar^2\omega_{32}^2(W_{31} + W_{32})} \left(\frac{p_{13}^2}{T_{13}} W_{32} - \frac{p_{23}^2}{T_{23}} W_{31} \right) N_0,$$

$$B = \frac{2}{\hbar^2\omega_{31}^2(W_{31} + W_{32})} \left(\frac{p_{13}^2}{T_{13}} W_{32} + \frac{p_{23}^2}{T_{23}} W_{31} \right),$$

$$g = 2\pi\omega_{21} \frac{p_{13}^2 \omega_{32} p_{23}^2}{\hbar\omega_{31}}.$$

The terms in the right hand sides of equations (1), which have the proportionality coefficient a , correspond to two-photon processes. The terms $2A(E_1^2 + E_2^2)$ and $2B(E_1^2 + E_2^2)N_{21}$ are related to the non-resonance effect of the fields E_1 and E_2 on the population of the working levels.

Equations (1) were obtained on the assumption that the second operational level is a metastable level, and it is assumed that the following conditions are fulfilled:

$$W_{13} \ll W_{31} + W_{32}; \quad W_{31} + W_{32} \gg \frac{\omega_n}{2Q_n} : \frac{2(E_1^2 + E_2^2)}{\hbar^2\omega_{31}^2} \left(\frac{p_{13}^2}{T_{13}} + \frac{p_{23}^2}{T_{23}} \right) \ll$$

$$\ll W_{31} + W_{32}; \quad (2)$$

$$\frac{4\pi p_{12}^2}{\hbar T_{12}\omega_m^2} N_{21} Q_n \ll 1.$$

It was shown in (Ref. 4) that the derivatives $\frac{dP_3}{dt}$ and $\frac{d\Phi}{dt}$ in the first and fourth equations of system (1) may be disregarded, if

$$\frac{\omega_n}{2Q_n}, W_{mn}, \frac{|P_{mn}|^2 E_1 E_2}{2\hbar^2 \omega_{mn}} \ll \frac{1}{T_{12}}. \quad (3)$$

Inequalities (2) and (3) will be fulfilled if

$$W_{13} \ll 10^8 \text{ sec}^{-1}, Q_n \gg 5 \cdot 10^6, E_1 E_2 \ll 18^8 \text{ CGS } \mathcal{E}. \quad (4)$$

$N_{21}^0 \ll 10^{25} \text{ cm}^{-3}$, and it was thus assumed that $W_{31} + W_{32} = 10^8 \text{ sec}^{-1}$;

$$\omega_n \approx 10^{15} \text{ sec}^{-1}; \omega_{31} \approx 5 \cdot 10^{15} \text{ sec}^{-1}; p_{13}^2 \approx p_{23}^2 \approx 10^{-36} \text{ DGS } \mathcal{E};$$

$$p_{12}^2 \approx 10^{-40} \text{ CGS } \mathcal{E}; T_{mn} \approx 10^{-10} \text{ sec}$$

When condition (4) is fulfilled, the system of equations (1) is simplified and assumes the following form:

$$\begin{aligned} \frac{dE_1^2}{dt} &= \left[\beta_1 \frac{E_2^2 N_{21}}{1 + (\Delta + \mathcal{E} N_{21})^2 T_{12}^2} - \frac{1}{\tau_1} \right] E_1^2, \\ \frac{dE_2^2}{dt} &= \left[\beta_2 \frac{E_1^2 N_{21}}{1 + (\Delta + \mathcal{E} N_{21})^2 T_{12}^2} - \frac{1}{\tau_2} \right] E_2^2, \\ \frac{dN_{21}}{dt} + \frac{N_{21} - N_{21}^0}{T_1} &= -\alpha \frac{E_1^2 E_2^2 N_{21}}{1 + (\Delta + \mathcal{E} N_{21})^2 T_{12}^2} + 2A(E_1^2 + E_2^2) - \\ &\quad - 2B(E_1^2 + E_2^2) N_{21}, \end{aligned} \quad (5)$$

where

$$\beta_n = \frac{16\pi p_{12}^2}{\hbar} T_{12} a^2 \omega_n; \alpha = \frac{16p_{12}^2}{\hbar^3} T_{12} a^2; \frac{1}{\tau_n} = \frac{\omega_n}{Q_n}; \text{ctg } \Phi \approx -(\Delta + \mathcal{E} N_{21}) T_{12}.$$

Equations (5) differ from the equations analyzed in (Ref. 2) by the fact that the phase relationships are taken into account. Following a procedure similar to that employed in (Ref. 2), we may select the incoherent pumping W_{13} in such a way that we may disregard the non-resonance effect of the fields E_1 and E_2 on the population of the operational levels. In the case of $W_{13} \gg W_{21}$ and $Q_n = 10^7$, this condition may be reduced to the following requirement

$$N_0 \ll 10^{22} \text{ cm}^{-3}. \quad (6)$$

When inequality (6) is fulfilled, the system of equations (5) may be written in the following form

$$\begin{aligned} \gamma \dot{v} + v &= 0 \\ \gamma \dot{u} + u &= \frac{u^2 - v^2}{[1 + (\Delta_1 - \lambda z)^2]} z, \end{aligned}$$

/41

$$\dot{z} + z - z_0 = \frac{v^2 - u^2}{1 + (\Lambda_1 - \lambda z)^2} z,$$

where

$$\begin{aligned} v &= \frac{1}{2} \left[E_1^2 \sqrt{\frac{\alpha \beta_2 T_1}{\beta_1}} - E_2^2 \sqrt{\frac{\alpha \beta_1 T_1}{\beta_2}} \right], \quad z = N_{21} \sqrt{\frac{\beta_1 \beta_2 \tau_1^2}{\alpha T_1}}, \\ u &= \frac{1}{2} \left[E_1^2 \sqrt{\frac{\alpha \beta_2 T_1}{\beta_1}} + E_2^2 \sqrt{\frac{\alpha \beta_1 T_1}{\beta_2}} \right], \quad z_0 = N_{21}^0 \sqrt{\frac{\beta_1 \beta_2 \tau_1^2}{\alpha T_1}}, \\ \tau_1 &= \tau_2; \quad \gamma = \frac{\tau_1}{T_1}, \quad \Delta_1 = \Delta T_{12}; \quad \lambda = + \frac{\kappa \sqrt{T_{12} T_1}}{4\pi \rho_{12} a \sqrt{Q_1 Q_2}}, \\ \dot{X} &= \frac{dX}{dt_1}, \quad t_1 = \frac{t}{T_1}. \end{aligned}$$

Three equilibrium configurations are possible in the system:

(a) $v = 0, u = 0, z = z_0,$

b) $v = 0, u' = \frac{z_0(1 + 2\lambda^2) + 2\Lambda_1\lambda - \sqrt{z_0^2 - 4\lambda\Lambda_1 z_0 - 4(1 + \Lambda_1^2 + \lambda^2)}}{2(1 + \lambda^2)},$

$$z' = \frac{z_0 - 2\Lambda_1\lambda + \sqrt{z_0^2 - 4\lambda\Lambda_1 z_0 - 4(1 + \Lambda_1^2 + \lambda^2)}}{2(1 + \lambda^2)},$$

c) $v = 0, u'' = \frac{z_0(1 + 2\lambda^2) + 2\Lambda_1\lambda + \sqrt{z_0^2 - 4\lambda\Lambda_1 z_0 - 4(1 + \Lambda_1^2 + \lambda^2)}}{2(1 + \lambda^2)},$

$$z'' = \frac{z_0 - 2\Lambda_1\lambda - \sqrt{z_0^2 - 4\lambda\Lambda_1 z_0 - 4(1 + \Lambda_1^2 + \lambda^2)}}{2(1 + \lambda^2)}.$$

(7)

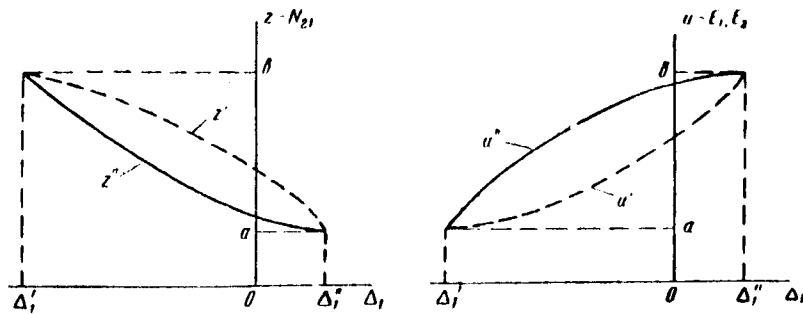


Figure 1

Dependence of Stationary Amplitudes of the Fields E_1 and E_2 and the Population Difference N_{21} on the Frequency Difference Δ_1 . The Branches u', z' of the Resonance Curves are Always Unstable; the Branches u'', z'' are Stable, if the Inequality Given in (Ref. 9) is Fulfilled.

The second and third equilibrium configurations exist only in the case of $z_0 \geq 2$ in the frequency difference range

$$\Delta_1' - \frac{\lambda z_0}{2} - \sqrt{(1 + \lambda^2) \left(\frac{z_0^2}{4} - 1 \right)} \leq \Delta_1 < \Delta_1' - \frac{\lambda z_0}{2} + \sqrt{(1 + \lambda^2) \left(\frac{z_0^2}{4} - 1 \right)}. \quad (8)$$

The dependence of u' , u'' and z' , z'' on the frequency difference Δ_1 is shown in Figure 1.

A study of the stability of the representative point motion close to the plane $v = 0$ [just as was done in (Ref. 2)] shows that the equilibrium state (7, a) is always a stable node. The characteristic equation for the two other equilibrium configurations has the following form

$$\gamma p^2 + \left[\gamma \frac{1 + \Lambda_1^2}{z^2} - \gamma(\lambda^2 - 1) - 1 \right] p + \left[\frac{1 + \Lambda_1^2}{z^2} - (1 + \lambda^2) \right] = 0.$$

The state of equilibrium (7, b) is always a saddle. The stability of the equilibrium configuration (7, c) changes as a function of the quantity z as follows:

$1 + \lambda^2 < \frac{1 + \Lambda_1^2}{z^2} < \frac{3 + \gamma(\lambda^2 - 1) - 2\sqrt{2 + \gamma(3\lambda^2 - 1)}}{\gamma}$ unstable node,

/43

$$\frac{3 + \gamma(\lambda^2 - 1) - 2\sqrt{2 + \gamma(3\lambda^2 - 1)}}{\gamma} < \frac{1 + \Lambda_1^2}{z^2} < \frac{\gamma(\lambda^2 - 1) + 1}{\gamma} \text{ unstable focus,}$$

$$\frac{\gamma(\lambda^2 - 1) + 1}{\gamma} < \frac{1 + \Lambda_1^2}{z^2} < \frac{3 + \gamma(\lambda^2 - 1) + 2\sqrt{2 + \gamma(3\lambda^2 - 1)}}{\gamma} \text{ stable focus,}$$

$$\frac{3 + \gamma(\lambda^2 - 1) + 2\sqrt{2 + \gamma(3\lambda^2 - 1)}}{\gamma} < \frac{1 + \Lambda_1^2}{z^2} \text{ stable node.}$$

If the third equilibrium state is stable

$$\frac{1 + \Lambda_1^2}{z^2} > \frac{\gamma(\lambda^2 - 1) + 1}{\gamma}. \quad (9)$$

rigid excitation of the fields E_1 and E_2 is possible in the system in the frequency difference range (8). In order to start a two-photon laser, it is necessary to supply it with a field E_2 (or E_1) $> E \sim u'$ from a foreign source.

If condition (9) is not fulfilled, then -- as was shown in (Ref. 2) -- an unstable limiting cycle exists in the system. In this case, a two-photon laser can emit only a single impulse in the frequency difference range (8) in the case of $z_0 \geq 2$.

In order that configurations 2 and 3 may exist, it is necessary that $N_0 \sqrt{W_{13}} \geq 5 \cdot 10^{21} \text{ cm}^{-3} \text{ sec}^{-1/2}$ (i.e., $z_0 \geq 2$). If $W_{13} = 10^6 \text{ sec}^{-1}$, then N_0 must be $> 5 \cdot 10^{18} \text{ cm}^{-3}$. The third equilibrium state is stable if $N_0 > 4 \cdot 10^{19} \text{ cm}^{-3}$.

Analysis of Non-Stationary Processes in a Two-Photon Laser in the Approximation of the Given Field

Let us assume that the process is incoherent and that the parameters of the system are such that the following inequality is fulfilled, besides the conditions (2) and (3)

$$\frac{N_{21}^0}{T_1} + 2A(E_1^2 + E_2^2) - \left[\frac{1}{T_1} + 2B(E_1^2 + E_2^2) \right] \ll \alpha E_1^2 E_2^2 N_{21}. \quad (10)$$

The occurrence of the term EN_{21} in the right hand sides of system of equations (5) is caused by the fact that, when an active substance is introduced into the resonator, its eigen frequency changes. When the non-stationary processes are analyzed, this effect is not taken into account. We shall assume that the frequency difference Δ_1 equals zero.

When (10) is fulfilled, the system of equations (5) acquires the following form

$$\begin{aligned} \frac{dS_1}{dt} &= B_1 S_1 S_2 N_{21} - \frac{S_1}{\tau_1}, \\ \frac{dS_2}{dt} &= B_1 S_1 S_2 N_{21} - \frac{S_2}{\tau_2} + f, \\ \frac{dN_{21}}{dt} &= -B_1 S_1 S_2 N_{21}, \end{aligned} \quad (11)$$

where $S_n = \frac{E_n^2}{\frac{\pi}{2} \hbar \omega_n}$ is the density of photons having the frequency ω_n , N_{21} - population difference of the operational levels; f - determines the intensity of the foreign source having the frequency ω_2 ; /44

$$B_1 = 4\pi^2 \omega_1 \omega_2 \rho_{21}^2 a^2 T_{12} \approx 4 \cdot 6 \cdot 10^{-28} \text{ cm}^6 \text{ sec}^{-1}.$$

Let us examine the case when the field E_2 is given from a foreign source with the intensity f , and the quality of the resonator is sufficiently small at the frequency ω_2 . We then have from the second equation in system in (11)

$$S_2 = \frac{\tau_2 f}{1 - B_1 S_1 \tau_2 N_{21}}. \quad (12)$$

Assuming that the condition $B_1 S_1 \tau_2 N_{21} \ll 1$ is fulfilled, we find that $S_2 \approx \tau_2 f = \text{const.}$

Integrating the third equation of system (11) for N_{21} under the initial condition $N_{21}(t = -\infty) = N_{21}^0$, we obtain

$$N_{21} = N_{21}^0 e^{-B_1 S_1 \int_{-\infty}^t S_1 dt}. \quad (13)$$

Then the first equation of system (11) assumes the following form after simple transformations

$$S_1 = \frac{dx_1}{dt} = N_{21}^0 [1 - e^{-\alpha x_1} - \alpha_1 x_1], \quad (14)$$

where the variable $x_1 = \int_{-\infty}^t S_1 dt$ is proportional to the energy of the radiation burst at the frequency ω_1 , $\sigma_1 = B_1 S_2$; $\alpha_1 = \frac{1}{\tau_1 N_{21}^0}$ (the occurrence of spontaneous radiation may be taken into account, by assigning the initial condition for S_1 : $S_1(-\infty) = S_{10}$).

The right hand side of equation (14) has one or two non-negative roots depending on the relationship between the parameters σ_1 and α_1 .

If $\frac{\sigma_1}{\alpha_1} < 1$, the right hand side equals zero only in the case of $x_1 = 0$.

This means that the system is not excited in the case of $S_2 < S_{20} = \frac{1}{B_1 \tau_1 N_{21}^0}$ --

i.e., there is a certain threshold value for the intensity of the outer source at the frequency ω_2 :

$$f_{\text{thresh}} = \frac{1}{B_1 \tau_1 \tau_2 N_{21}^0} \frac{S_{20}}{\tau_2}. \quad (15)$$

In the case of $\frac{\sigma_1}{\alpha_1} = \frac{S_2}{S_{20}} \geq 1$, the right hand side of (14) has two non-negative roots:

$$x_1 = 0 \text{ and } x_1' \approx -\frac{2(\sigma_1 - \alpha_1)}{\sigma_1^2}. \quad (16)$$

When equation (14) is analyzed, it may be readily seen that the root $x_1 = 0$ is unstable; x_1' is a stable root. It determines the energy of the radiation burst at the frequency ω_1 when there is a small increase in the excitation threshold (15).

If $\frac{\sigma_1}{\alpha_1} \gg 1$, then the roots equal

$$x_1 = 0 \text{ and } x_1' \approx \frac{1}{\alpha_1}. \quad (17)$$

When there is a significant increase in the excitation threshold, the energy of the radiation burst x_1 is only determined by the properties of the system, and does not depend on the intensity f of the outer source. /45

Equation (14) may be integrated in two limiting cases: $\frac{\sigma_1}{\alpha_1} \geq 1$ and

$\frac{\sigma_1}{\alpha_1} \gg 1$. Figure 2 shows the dependence of $S_1 = \frac{dx_1}{dt}$ on x_1 . Curve 1 was compiled for the case $\frac{\sigma_1}{\alpha_1} \geq 1$, and curve 2 was compiled for case $\frac{\sigma_1}{\alpha_1} \gg 1$.

In the case of $\frac{\sigma_1}{\alpha_1} \geq 1$, equation (14) may be integrated with expansion of $e^{-\sigma_1 x_1}$ in series:

$$e^{-\sigma_1 x_1} = 1 - \sigma_1 x_1 + \frac{\sigma_1^2 x_1^2}{2} - \dots$$

Then the solution of equation (14) assumes the following form

$$\frac{x'_1 - x_1}{x_1} = e^{-\frac{4t}{t_0}},$$

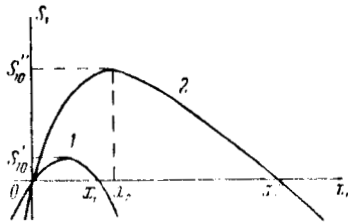


Figure 2

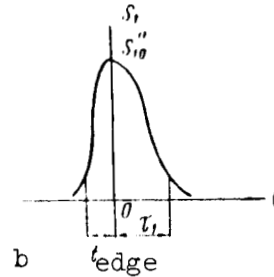
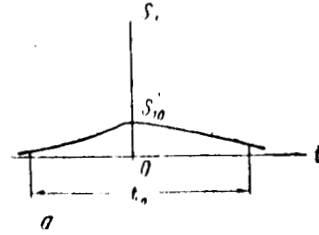


Figure 3

Dependence of the Photon Density S_1 of Frequency ω_1 on $x_1 = \int_{-\infty}^t S_1 dt$;

x_1 is Proportional to the Energy

of the Radiation Burst. Curve 1 was Compiled for the Case $\sigma_1/\alpha_1 \geq 1$;

2 - for $\sigma_1/\alpha_1 \gg 1$.

Form of the Radiation Burst at the Frequency ω_1 . a - in the Case $\sigma_1/\alpha_1 \geq 1$; b - in the Case $\sigma_1/\alpha_1 \gg 1$.

We thus have

$$S_{10} = S'_{10} \operatorname{ch}^{-2} \left[\frac{2t}{t_0} \right]. \quad (18)$$

The radiation burst at the frequency ω_1 has a symmetrical form in the case of $\frac{\sigma_1}{\alpha_1} \geq 1$ (Figure 3, a). Its duration $t_0 = \frac{8}{\sigma_1^2 x'_1 N_{21}^0}$, amplitude $S'_{10} = \frac{\sigma_1^2 x'_1 N_{21}^0}{8}$, and energy x'_1 [relationship (16)] depend essentially on the intensity f of the outer source.

The number of active atoms decreases from N_{21}^0 in the case of $t = -\infty$ to $N_{21}(t = +\infty) = N_{21}^0 e^{-\sigma_1 x'_1}$ when there is a radiation burst:

$$N_{21} = N_{21}^0 e^{-\frac{\sigma_1 x_1}{1 + \exp\left|\frac{4t}{t_0}\right|}}$$

In the case $\frac{\sigma_1}{\alpha_1} \gg 1$, curve 2 in Figure 2 may be approximated in the form

$$\text{for } -\infty < x_1 < x_2 = -\frac{1}{\sigma_1} \ln \frac{\sigma_1}{\alpha_1}, \quad (19) \quad \underline{/46}$$

$$\frac{dx_1}{dt} = N_{21}^0 (1 - e^{-\sigma_1 x_1})$$

$$\text{and for } x_2 < x_1 < +\infty \quad \frac{dx_1}{dt} = N_{21}^0 (1 - \alpha_1 x_1). \quad (20)$$

Equations (19) and (20) may be readily integrated, and their solutions have the following form

$$S_1 = \frac{N_{21}^0}{1 + \left(\frac{N_{21}^0}{S_{10}^0} - 1\right) e^{-\sigma_1 N_{21}^0 t}} \quad -\infty < t \leq 0, \quad (21)$$

$$S_1 = S_{10}^0 e^{-\alpha_1 N_{21}^0 t} \quad 0 \leq t < +\infty, \quad (22)$$

where

$$S_{10}^0 = N_{21}^0 \left(1 - \frac{\alpha_1}{\sigma_1} - \frac{\alpha_1}{\sigma_1} \ln \frac{\sigma_1}{\alpha_1}\right).$$

For large intensities of the outer source at the frequency $\omega_2 \left(\frac{\sigma_1}{\alpha_1} \gg 1\right)$, the radiation burst has a non-symmetrical form (Figure 3, b). The duration of its leading edge is determined by the intensity of the source

$$t_{\text{edge}} = \frac{1}{B_1 N_{21}^0 \tau_2 f}; \quad (23)$$

the duration of the trailing edge depends on the quality of the resonator at the frequency ω_1

$$t_{\text{edge}} = \tau_1 = \frac{\omega_1}{Q_1}. \quad (24)$$

The amplitude of the impulse

$$S_{10}^0 = N_{21}^0 \left(1 - \frac{\alpha_1}{\sigma_1} - \frac{\alpha_1}{\sigma_1} \ln \frac{\sigma_1}{\alpha_1}\right) \quad (25)$$

strives to N_{21}^0 in the case of $\frac{\sigma_1}{\alpha_1} \rightarrow \infty$.

In the case of a radiation burst, the population difference decreases according to

$$N_{21} = \frac{N_{21}^0}{1 + \left(\frac{N_{21}^0}{S_{10}^0} - 1\right) e^{-\sigma_1 N_{21}^0 t}} \quad -\infty < t \leq 0, \quad (26)$$

$$N_{21} = N_{21}^0 \left(1 - \frac{S_{10}^*}{N_{21}^0}\right) e^{-\frac{\alpha_1}{\alpha_1} (1 - e^{-\alpha_1 N_{21}^0 t})}, \quad 0 \leq t < +\infty. \quad (26)$$

The expressions obtained (18), (21), (22) for the radiation burst characterize the transitional process to a stationary regime [which was described in (Ref.2, 3,)] in a two-photon laser in the approximation of the given field. Let us estimate the values of the fields E_1 , E_2 under which condition (10) is fulfilled. We shall assume that

$$\left. \begin{aligned} \omega_{21} &= 1.8 \cdot 10^{15} \text{ sec}^{-1}; \omega_2 = 1.2 \cdot 10^{15} \text{ sec}^{-1}; \omega_1 = 0.6 \times 10^{15} \text{ sec}^{-1}; Q_1 = 10^7; \\ a &= 4 \cdot 10^{-5} \text{ CGSE}; p_{12}^2 = 10^{-40} \text{ CGSE}; N_{21}^0 = 10^{20} \text{ cm}^{-3}; \omega_{31} = 2.5 \cdot 10^{15} \text{ sec}^{-1}; \\ p_{13}^2 &= p_{22}^2 = 10^{-36} \text{ CGSE}; B_1 = 4.6 \cdot 10^{-28} \text{ cm}^{-6} \text{ sec}. \end{aligned} \right\}$$

With allowance for expressions (13) and (14), for N_{22} and S_1 inequality (10) is equivalent to the requirement: $N_0 \ll 10^{22} \text{ cm}^{-3}$ and /47

$$e^{-\alpha_1 N_0} = 1 - \alpha_1 T_1 N_{21}^0 (1 - e^{-\alpha_1 N_{21}^0 t}). \quad (27)$$

It may be readily seen that inequality (27) is only fulfilled for the central portion of the radiation burst S_1 , if $\frac{T_1}{\tau_1} >> \frac{\sigma_1 + \alpha_1}{\sigma_1 - \alpha_1}$. In order to describe

the excitation process of the system ($t \rightarrow -\infty$) and its approach to a stationary regime ($t \rightarrow +\infty$), we must take into account spontaneous radiation. For the given system parameters; the condition of laser excitation is fulfilled if the intensity of the outer source is such that $S_2 > S_{20} = 1.3 \cdot 10^{15} \text{ cm}^{-3}$. Let

$S_2 = 1.43 \cdot 10^{15} \text{ cm}^{-3}$, then $\sigma_1 = 0.6 \cdot 10^{-12} \text{ cm}^{-3} \text{ sec}$; $\alpha_1 = 0.6 \cdot 10^{-12} \text{ cm}^{-3} \text{ sec}$; $\frac{\sigma_1}{\alpha_1} = 1.1$. The radiation burst has the following parameters: energy of the

burst $\epsilon_1' = \frac{h\omega_1 c}{8} x_1' = 42 \frac{\text{Joule}}{\text{cm}^2}$, duration $t_0 = 0.66 \mu \text{ sec}$, and the peak strength $P_{10}' = \frac{h\omega_1 c}{8} S_{10}' = 60 \frac{\text{mgm}}{\text{cm}^2}$. If $S_2 = 13 \cdot 10^{15} \text{ cm}^{-3}$, then $\sigma_1 = 6 \cdot 10^{-12} \text{ sec} \cdot \text{cm}^{-3}$; $\alpha_1 = 0.6 \cdot 10^{-12} \text{ cm}^{-3} \text{ sec}$; $\frac{\sigma_1}{\alpha_1} = 10$, and the parameters of the

radiation burst in this case are calculated according to formulas (17), (23-25): energy of the burst $\epsilon_1'' = 250 \text{ Joule/cm}^2$, duration of its leading edge

$t_{\text{edge}} = 1.7 \cdot 10^{-9} \text{ sec}$, duration of the trailing edge $t_{\text{edge}} = \tau_1 = 1.7 \cdot 10^{-8} \text{ sec}$, peak strength $P_{10}'' = 12.5 \frac{\text{gigawatt}}{\text{cm}^2}$.

If $\tau_2 = 10^{-5} \text{ sec}$, then condition (12) is fulfilled up to $S_1 \ll 2 \cdot 10^{18} \text{ cm}^{-3}$. Inequalities (4) are also fulfilled. Since the duration of the laser pulse

(from the outer source) is on the order of $1 \mu \text{ sec}$, then it may be assumed that S_2 is constant for $1 \mu \text{ sec}$. Thus, a certain initial number of photons S_{20} at the frequency ω_2 [formula (15)] is necessary for the excitation of a two-photon laser. When the condition of self-excitation is fulfilled in the approximation of the given field E_2 , the system generates an impulse of the frequency ω_1 . The form of the impulse depends essentially on the relationship between the parameters of the system and the field E_2 .

An approximate analysis of non-stationary processes in a two-photon laser in the case of a self-consistent field is performed in (Ref. 4-5).

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